Analysis of the Lens-Redshift Probability Distribution for Constraining Cosmological Parameters

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Abstract

This study revisits the lens-redshift probability distribution method using galaxy-scale gravitational lensing data. Two datasets, Sample A (161 systems) and Sample B (74 systems), characterized by distinct Einstein radius and redshift distributions, are analyzed under the ΛCDM, wCDM, and w_0w_aCDM models. The method employs a singular isothermal sphere model and utilizes velocity dispersion functions (VDF) from prior studies. Results indicate that cosmological constraints are insensitive to VDF variations but depend heavily on the data distribution. Combining Sample A with cosmic microwave background, baryon acoustic oscillations, and type Ia supernovae data tightens constraints toward $w = -1$, aligning with ΛCDM. While the method's robustness is limited by sample biases, future large-scale surveys like LSST and Euclid—incorporating lensing by late-type galaxies—are expected to improve accuracy and provide a more representative model of the Universe.

Background

- The ΛCDM model is the cosmological model most consistent with observations to date (e.g., [1–3]). Nevertheless, it faces serious challenges on both small and large scales.
- Strong gravitational lensing offers an independent probe free from the assumptions of distance ladders or CMB fluctuations.
- One effective approach to constraining cosmological parameters is the lens redshift distribution method [4]. However, this method is highly sensitive to sample selection biases.
- We revisit this method using recent galaxy-scale strong lens data, comparing three different cosmological models to derive tighter constraints on the nature of dark energy.

The Lens-Redshift Probability Distribution

The optical depth for lensing, τ, which quantifies the probability of a lensing event, is given by [4, 5]:

$$\frac{d\tau}{dz_l} = n\left(\theta_E, z_l\right)\left(1+z_l\right)^3 \; S\,\frac{c\,dt}{dz_l}.$$

Velocity Dispersion Function

The comoving number density $n(\theta_E, z_l)$ is derived from the velocity dispersion function (VDF):

$$n(\theta_E, z_l) = \frac{\phi(\sigma)d\sigma}{d\theta_E}.$$

We adopt the modified Schechter [6] form of the VDF for early-type galaxies [7]:

$$\phi(\sigma)d\sigma = \phi_* \left(\frac{\sigma}{\sigma_*}\right)^{\alpha} \exp\left[-\left(\frac{\sigma}{\sigma_*}\right)^{\beta}\right] \frac{\beta}{\Gamma\left(\alpha/\beta\right)}\frac{d\sigma}{\sigma}.$$

Table 1. The parameters of the VDF.

Reference	$\phi_*(10^{-3}\,\mathrm{Mpc}^{-3})$	$\sigma_*\,(\mathrm{km/s})$	α	β
B+10 [8]	1.79 ± 0.1	166.5 ± 17.02	2.54 ± 0.41	2.93 ± 0.33
C10 [9]	4.46	217	0.85	3.72
C+07 [10]	2.74	161 ± 5	2.32 ± 0.10	2.67 ± 0.07

Data Selection

The first dataset (Sample A) from C+19 [11] consists of 161 lens systems. These systems are ETGs with E/S0 morphology and possess no significant substructures or nearby massive companions. The second dataset (Sample B) was compiled from three main sources: SuGOHI, SQLS, and the GLQ Database. Sample B includes 74 lens systems with Einstein radii $\theta_E \leq 2''$.

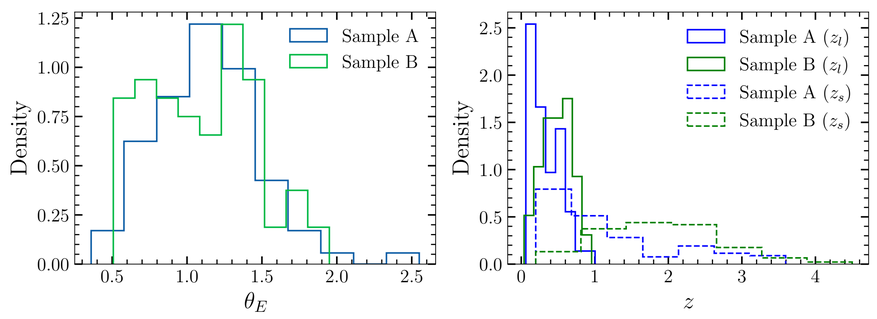


Figure 1. Distribution of data samples in Einstein radius (left) and redshift (right).

VDF Parameter Variations

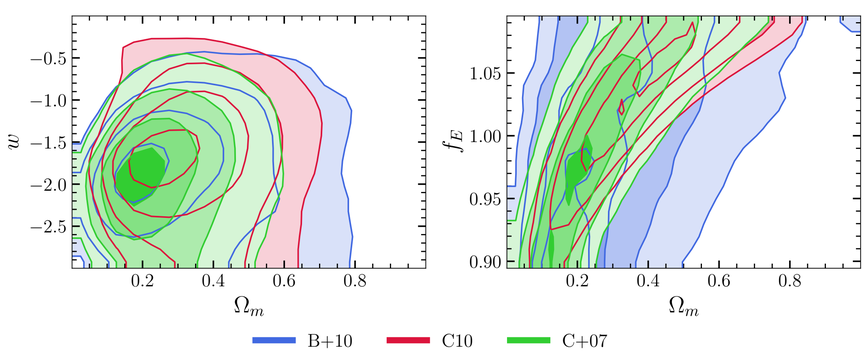


Figure 2. Constraints on cosmological parameters for three different VDF parameters.

Data Sample and Cosmological Model Variations

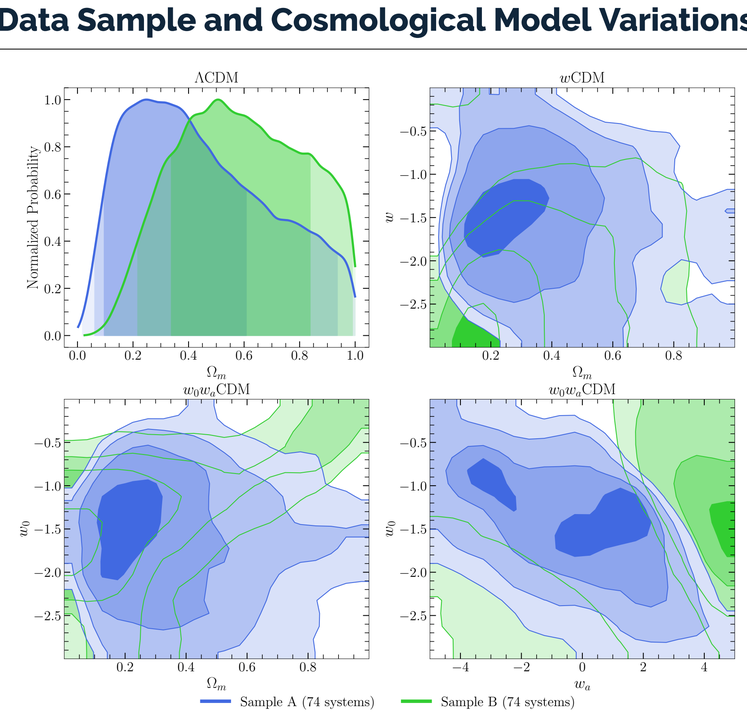


Figure 3. Constraints on cosmological parameters from two different samples across three cosmological models.

Dark Energy Constraints

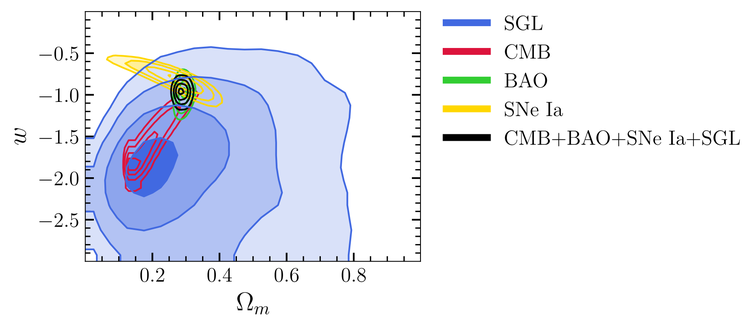


Figure 4. Constraints on cosmological parameters combined with other probes for the wCDM model.

Table 2. Cosmological parameter constraints from SGL and in combination with other probes.

Cosmological Model	Data	Ω_m	w_0	w_a
ΛCDM	SGL	$0.42^{+0.28}_{-0.19}$	–	–
wCDM	SGL	$0.33^{+0.30}_{-0.17}$	$-1.80^{+0.77}_{-0.69}$	–
wCDM	CMB + BAO + SNe Ia + SGL	$0.29^{+0.02}_{-0.02}$	$-0.96^{+0.09}_{-0.10}$	–
w_0w_aCDM	SGL	$0.32^{+0.31}_{-0.19}$	$-1.79^{+0.83}_{-0.75}$	$-0.29^{+3.21}_{-3.27}$

Conclusions

- The lens-redshift probability distribution method shows limited robustness due to strong data dependence, while velocity dispersion parameter changes have negligible impact.
- Independently, the model supports phantom dark energy ($w < -1$) and broad w_0w_aCDM constraints. However, combining with CMB, BAO, and SNe Ia tightens constraints, favoring ΛCDM ($w \approx -1$).
- Larger datasets (e.g., LSST, Euclid) combined with late-type galaxy lensing will further improve future results.

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