Searching for Feature Signals in Gravitational Waves From Core-Collapse Supernovae Using the Hilbert-Huang Transform

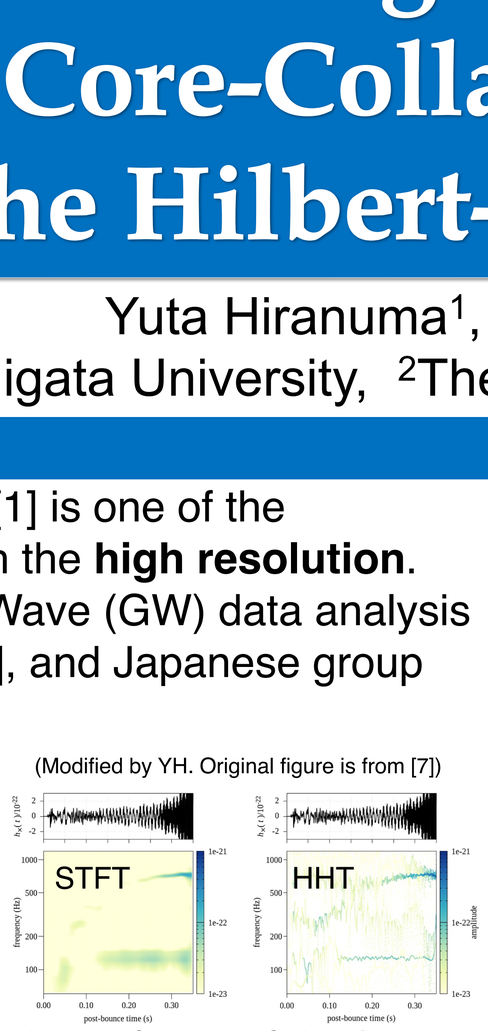
Yuta Hiranuma[1], Ken-ichi Oohara[1,2]
([1]Niigata University, [2]The Open University of Japan)
E-mail: yuta@astro.sc.niigata-u.ac.jp

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1. Introduction & Aim

- The **Hilbert-Huang Transform (HHT)** [1] is one of the **time-frequency** analysis methods with the **high resolution**.
- The HHT was applied to Gravitational Wave (GW) data analysis first by Camp *et al.* [2], Stroeer *et al.* [3], and Japanese group follows them.
  - Binary neutron star merger [4], [9]
  - Quasinormal modes in ringdown [5]
  - Core-collapse supernova (CCSN) — SASI [7]

(Modified by YH. Original figure is from [7])

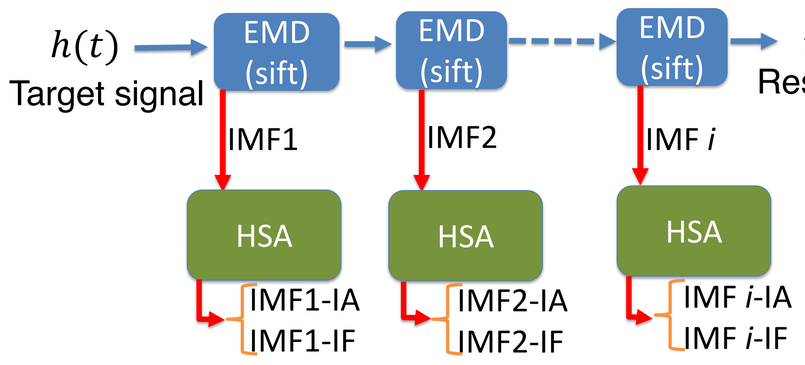


- In this work, We apply the HHT to GW signal from a failed CCSN and change parameters of the empirical mode decomposition.

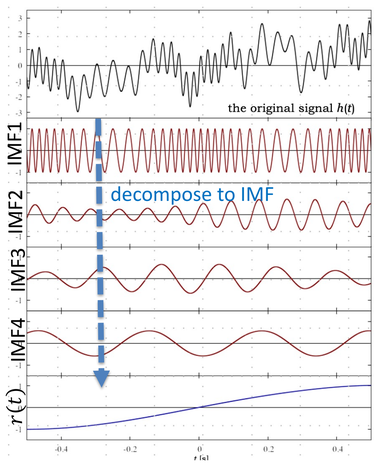
2. Hilbert-Huang Transform (HHT)

◆ Overview of the HHT

The HHT consists of two parts
✓ **Empirical Mode Decomposition (EMD)**
✓ **Hilbert Spectral Analysis (HSA)**



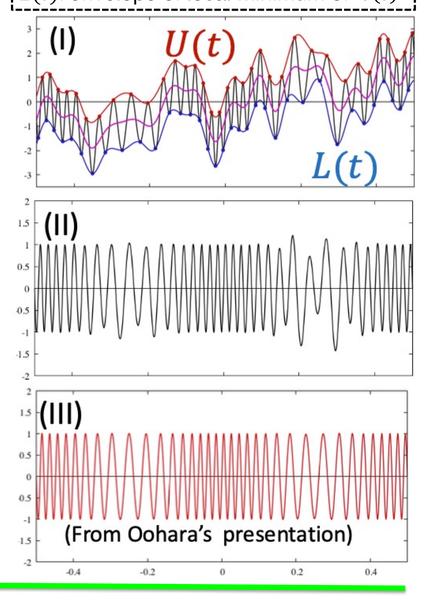
Intrinsic Mode Function (IMF)
Instantaneous Amplitude (IA)
Instantaneous Frequency (IF)



(From Oohara's presentation)

Empirical Mode Decomposition (EMD)

$U(t)$: envelope of local maximum of $c(t)$
$L(t)$: envelope of local minimum of $c(t)$



(From Oohara's presentation)

✓ EMD sifts original signal, like a high pass filter.

- set $h_0(t) \leftarrow h(t)$

1. $c(t) \leftarrow h_0(t)$
2. $m(t) \leftarrow (U(t) + L(t))/2$ …(I)
3. $c_{\mathrm{new}}(t) \leftarrow c(t) - m(t)$ …(II)

if a stoppage criterion is satisfied *then*
  get IMF $= c(t)$, …(III)
  $h_0(t) \leftarrow h_0(t) - c(t)$,
  goto 1. ➔ Search next IMF
else
  $c(t) \leftarrow c_{\mathrm{new}}(t)$,
  goto 2.
end if

"EMD stoppage criterion"
$$\frac{\sum_i |m(t_i)|^2}{\sum_i |c(t_i)|^2} < \boldsymbol{\varepsilon}$$

Hilbert Spectral Analysis (HSA)

✓ Get IA, IF of each IMF ($x(t)$).

$$z(t) = x(t) + i\mathcal{H}[x(t)] = a(t)e^{i\theta(t)}$$

$$\left(\mathcal{H}[x(t)] = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(t)}{t-\tau} d\tau\right)$$
(PV: Principal value)

$a(t) = \sqrt{x^2(t) + \mathcal{H}[x(t)]^2}$: **Instantaneous Amplitude (IA)**
$f(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$: **Instantaneous Frequency (IF)**

3. Ensemble Empirical Mode Decomposition (EEMD)

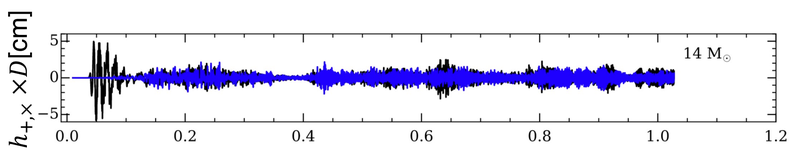
Add a white Gaussian noise with a standard deviation σ_{e} to the original data $h(t)$.
($\sigma_{\mathrm{e}} = \sigma_h \times \sigma_{\mathrm{eemd}}$ with a pre-determined $\boldsymbol{\sigma_{\mathrm{eemd}}}$)($\sigma_h$:standard deviation of $h(t)$)

(i) Decompose the data with the white noise into IMFs.
(ii) Repeat steps (i) and (ii) many times but with different white Gaussian noise series at each time.
(iii) Obtain the **ensemble means** for the series of the **obtained IMFs**. The number of ensemble trials, $\boldsymbol{N_{\mathrm{eemd}}}$, has to be large enough.

(E)EMD parameters
$\boldsymbol{\varepsilon}$: stoppage criterion
$\boldsymbol{N_{\mathrm{eemd}}}$: ensemble number
$\boldsymbol{\sigma_{\mathrm{eemd}}}$: injection noise parameter

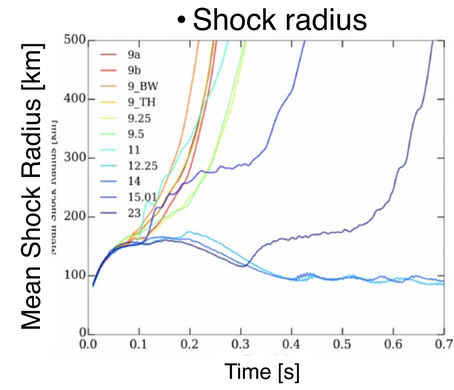
4. Target Signal & Set Up

Vartanyan et al. (2023) [8]
non-rotating 3D CCSN simulations
- Non-exploding model ($14M_\odot$), h_+
  ✓SASI active: ~100 Hz



(Modified by YH. Original FIG. is from [8]. Cross (blue) and plus (black) polarizations. GW strain data are from https://www.astro.princeton.edu/~burrows/gw.3d.new/ .)

standing accretion shock instability (SASI)

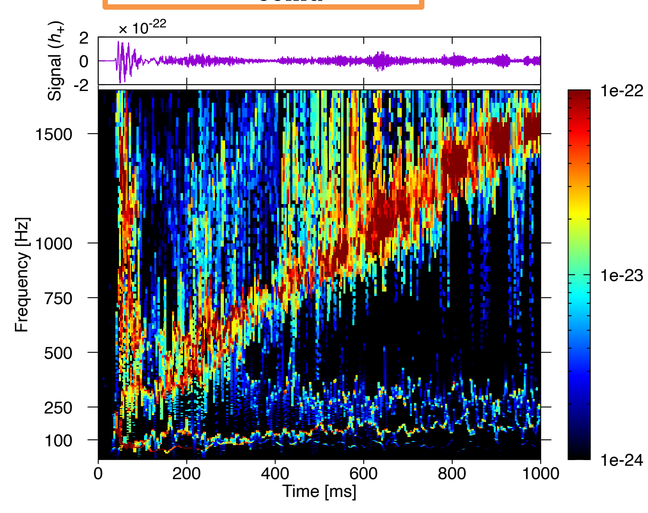


(Mod. by YH. Original Fig. is from [8])

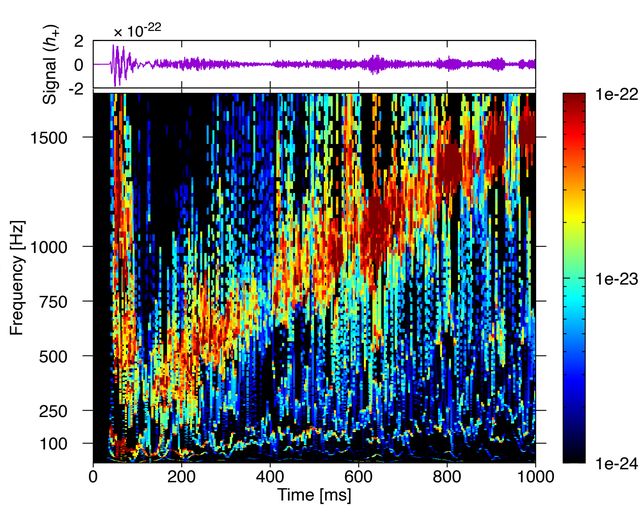
5. Results *Preliminary*

- EEMD parameters list
  - ➢ Ensemble number: $N_{\mathrm{eemd}} = 10^5$ (fix)
  - ➢ Stoppage criterion: $\varepsilon = 0.1, 10^{-3}$
  - ➢ Injection noise parameter: $\sigma_{\mathrm{eemd}} = 10, 10^{-1}$

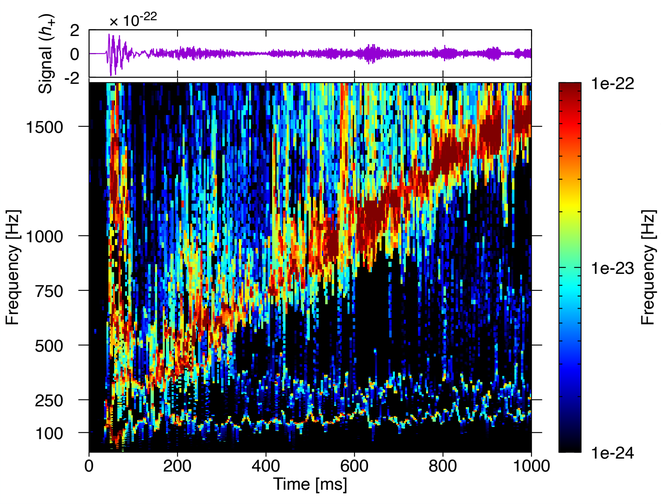
$\varepsilon = 0.1, \sigma_{\mathrm{eemd}} = 10$



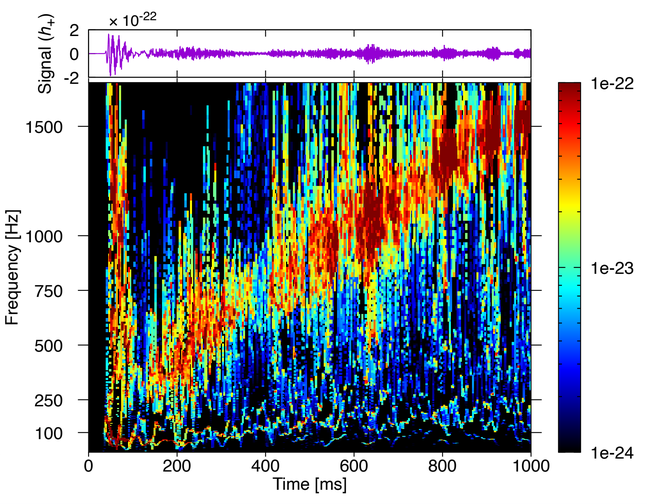
$\varepsilon = 0.1, \sigma_{\mathrm{eemd}} = 10^{-1}$



$\varepsilon = 10^{-3}, \sigma_{\mathrm{eemd}} = 10$



$\varepsilon = 10^{-3}, \sigma_{\mathrm{eemd}} = 10^{-1}$



6. Summary

✓ Changed some EEMD parameters and applied the HHT to GW signal from the CCSN.
→ When large noise injected ($\sigma_{\mathrm{eemd}} = 10$), more clearly, SASI possible signal appeared.

Acknowledgements

YH used HHT code of `KAGALI` (KAGRA Algorithmic Library) and the GW signal data from https://www.astro.princeton.edu/~burrows/gw.3d.new/ .

Reference

[1] N. E. Huang, and S. S P Shen, *Hilbert-Huang Transform and Its Applications*, World Scientific (2005)
[2] J. B. Camp, J. K. Cannizzo, and K. Numata, Phys. Rev. D **75**, 061101 (2007)
[3] A. Stroeer, J. K. Cannizzo, and J. B. Camp, Phys. Rev. D **79**, 124022 (2009)
[4] M. Kaneyama, K. Oohara, H. Takahashi, Y. Sekiguchi, H. Tagoshi, and M. Shibata, Phys. Rev. D **93**, 1203010 (2016)
[5] K. Sakai, K. Oohara, H. Nakano, M. Kaneyama, and H. Takahashi, Phys. Rev. D **96**, 044047 (2017)
[6] T. Kuroda, K. Kotake, and T. Takiwaki, ApJ, **827**, L14 (2016)
[7] M. Takeda, Y. Hiranuma, N. Kanda, K. Kotake, T. Kuroda, R. Negishi, K. Oohara, K. Sakai, Y. Sakai, T. Sawada, H. Takahashi, S. Tsuchida, Y. Watanabe, and T. Yokozawa, Phys. Rev. D **104**, 084063 (2021)
[8] D. Vartanyan, A. Burrows, T. Wang, M. S. B. Coleman, and C. J. White, Phys. Rev. D **107**, 103015 (2023).
[9] I. Yoda, K. Oohara, H. Takahashi, and K. Sakai, PTEP, **2023**, 083E01 (2023)